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Steven L. Puterbaugh Arthur J. Wennerstrom

Technology Branch Turbine Engine Division

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### FOREWORD

This report contains the results of an effort to modify the loss reestimation procedure utilized in the UD0300 axial compressor design computer program. A three-dimensional shock analysis was added in order to improve the shock loss prediction accuracy. The work was done by the Compressor Research Group, Technology Branch, Turbine Engine Division, Aero Propulsion Laboratory, Air Force Wright Aeronautical Laboratories at Wright-Patterson Air Force Base, Ohio. All debugging and code evaluation was done on an IBM 370 system. The effort was conducted by S. L. Puterbaugh and Dr A. J. Wennerstrom from August 1980 to August 1981 under Project 2307, Task S1, Work Unit 27, "Turbomachinery Fluid Mechanics."

The code is included in the latest version of UD0300 and is available upon request to AFWAL/POTX.

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## LIST OF SYMBOLS

В	vector tangent to blade surface and computing station surface
F <sub>p</sub>	vector normal to the blade surface
Fs	vector tangent to a blade in a stream surface
i <sub>m</sub>	unit vector; see Figure 4
θţ	unit vector; see Figure 4
k <sub>n</sub>	unit vector; see Figure 4
1	coordinate along a computing station
m	meridional intrinsic coordinate
n	normal intrinsic coordinate
r	radius from axis of symmetry
β	relative flow angle
Υ	computing station lean angle
ε	blade lean angle
Θ	angular coordinate about axis of symmetry
ø	Stream surface nitch angle

### SECTION I

## INTRODUCTION

This report describes the use of a three-dimensional shock model in the estimation of shock losses in a compressor rotor blade row, based on the observation of D. C. Prince, Jr., in Reference 1. In essence, the model exploits the three-dimensional aerodynamic sweep associated with a shock surface produced by the spanwise distribution of a series of normal shocks located on concentric stream surfaces. The aerodynamic sweep reduces the Mach number normal to the shock surface, hence, reducing the strength of the shock.

An illustrative example is presented and discussed in Section IV of the report.

#### SECTION II

#### BACKGROUND

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As stated in the introduction, the shock model which was used in the new loss estimation procedure was discussed in a paper by David C. Prince, Jr. in the Journal of Aircraft (Reference 1). The paper includes a number of experimental observations which illustrate two significant points concerning the shock structure present in transonic/supersonic blade rows. The two points concern the orientation of the passage shock which extends from the leading edge of a blade to the suction surface of the adjacent blade. First, when the rotor is operating at the design point, the shock lies approximately axially (Figure 1); and secondly, when the rotor is operating at relatively high back pressures, the shock lies approximately normal to the flow on a stream surface (Figure 2). This second case represents operation at or near peak efficiency.

The consistency with which these phenomena occur allows for the modification of the existing shock loss calculation procedure found in UD0300 (Reference 2) based on their characteristics. The result is a more realistic and, happily, lower estimation of the loss incurred through the presence of shocks in the blade row. The original method of calculation of shock loss found in UD0300 will now be discussed.

Losses are incorporated into the axisymmetric flow solution in UD0300 through relative total pressure loss coefficients. The loss coefficient is made up of two components; the loss due to diffusion in the profile boundary layer, and the loss due to the presence of shocks.

The shock loss was calculated assuming a normal shock is present in the blade passage, much like that which exists when at or near peak efficiency. A shock Mach number was calculated by averaging the inlet Mach number and a suction surface Mach number calculated by a Prandtl-Meyer expansion (or compression) through an input expansion (or compression) angle (Figure 2). The total pressure loss across the shock was then calculated using standard normal shock parameter ratio equations at the shock Mach number.

This calculation produced a pessimistic prediction at low aspect ratios, which became even more so at higher tip speeds. It was felt that this was an unrealistic model which severely limited attempts to explore higher tip speed designs.

#### SECTION III

#### THEORY AND METHODS

The shock configuration associated with the maximum efficiency operating point was used as the basis of the model. This was done due to the geometric simplicity of that configuration in that the shock impingement point on the suction surface could more easily be determined. A comparison of the two configurations (Figures 1 and 2) illustrates the rationale for this decision. Since the maximum efficiency configuration was used, a design which is developed utilizing this model must, obviously, be evaluated in that light.

The shock surface mentioned previously is created by a spanwise distribution of normal passage shocks (Figure 3). In low aspect ratio blading, like that in Figure 3, the aerodynamic sweep of the shock surface at the adjacent suction surface is highly exaggerated due to twist in the blade. It is the combination of this sweep and the leading edge sweep which is used in calculating the magnitude of the normal component of the local relative Mach number which is incorporated into the shock loss computation. This modified shock Mach number reduces the total pressure ratio across the shock, thus reducing the shock loss relative to the previous method.

The stream surface intersection of two adjacent blades at each streamline is developed onto a flat surface normal to the flow direction in order to determine the suction surface impingement point of the shock. The impingement point is the intersection of the suction surface of the appropriate blade and a line representing the shock which is perpendicular to the inlet flow and intersecting the leading edge of the adjacent blade. The relative Mach number at the shock impingement is calculated by a Prandtl-Meyer expansion (or compression) through a turning angle equal to the difference of the inlet relative flow angle and the suction surface angle at the impingement point.

The aerodynamic sweep is calculated for the leading edge and the locus of shock impingement points on the suction surface. The sweep angle,  $\nu$ , is a function of several blade configuration parameters and is given by:

 $v = \cos^{-1} (\cos \beta \cos \epsilon \sin (\theta - \gamma) - \sin \beta \sin \epsilon) - 90^{\circ}$  where

- $\beta$  = relative flow angle
- ε = blade lean angle
- # = stregaline slope angle
- $\gamma$  = computing station lean angle

The definition and orientation of these angles are given in Figure 4. The inlet and suction surface Mach numbers are then modified by their associated aerodynamic sweep angles.

Since total pressure ratio across a normal shock is not a linear function of Mach number, a simple three-point Simpson's Rule integration of shock pressure ratio is performed along the shock for streamlines at greater radii than the sonic radius. Leading edge, mid-channel, and suction surface values of sweep and Mach number are used for the integration. The mid-channel value of sweep and Mach number is an average of the two end values. The integrated shock pressure ratio is then incorporated into the shock loss component of the relative total pressure loss coefficient.

A finite value of shock loss is present in streamtubes below the sonic radius due to a transonic bubble on the suction surface. The following procedure was developed to address this situation. When the sweep corrected midchannel Mach number is greater than 1.0, the shock loss is calculated based on the product of the mid-channel value and the inlet relative Mach number. If the product is less than 1.0, the shock loss is set to 0.0.

#### SECTION IV

#### OPTION IMPLEMENTATION

The incorporation of the three-dimensional sweep model into the loss re-estimation calculation is triggered by setting the value of NDEL at the appropriate computing station less than zero. The previous shock loss calculation procedure will be used for positive values of NDEL. The only other input data requirement is that the number of blade design passes must be greater than one.

A table of values entitled "Shock Surface Sweep Calculation Parameters" is printed at the end of the flow solution. The table consists of streamline number, inlet radius, leading edge sweep angle, suction surface sweep angle, shock pressure ratio, and calculated Prandtl-Meyer expansion angle.

The flow of the program when executed with the three-dimensional shock model is as follows:

- During the first design pass, the losses are not re-estimated.
   This is done so that a converged solution is used as a base for the re-estimation pass(es).
- The shock surface sweep values are calculated for use in the next design pass. The calculation is done in subroutine UD0331, which is called from subroutine UD03AB.
- 3. After the first design pass, the aerodynamic sweep angles calculated on the previous pass are used in determining the relative total pressure loss coefficients to be used in that pass.

#### SECTION V

#### TRIAL EXAMPLES

In order to substantiate the credibility of the geometric calculations, four test cases were conceived. Each was designed to produce zero work and each was without any annular convergence. The influence of blade lean and leading edge meridional sweep on the aerodynamic sweep angle was to be investigated. The four cases were:

Case I - zero LE sweep and zero LE lean

Case II - 30° LE sweep and zero LE lean

Case III - zero LE sweep and 30° LE lean at the tip

Case IV - 30° LE sweep and 30° LE lean at the tip

The results of these example cases are listed in Tables 1 thru 4. All trends of the parameter were what was expected and, therefore, gave credibility to the geometric calculations.

The test of credibility of the entire model was conducted in an evaluation of the USAF/AFAPL HTF compressor stage (Reference 3). Since this stage has been fabricated and tested, experimental results were available which offered an excellent opportunity to evaluate the new shock loss model in the light of a design result versus experimental result comparison.

The experimental results revealed that the isentropic efficiency curve peaked approximately 5 percentage points higher than the design efficiency. When the design was rerun with the three-dimensional shock model included, all else being equal, an increase of approximately 3.75 percentage points was noted. Although this close correlation on the basis of one example could be fortuitous, it was viewed as a highly promising result.

## SECTION VI

### CONCLUSIONS

The shock loss calculation technique reported on herein appears to give a more realistic evaluation of the losses present in a transonic/supersonic compressor rotor of low aspect ratio. The technique was evaluated both geometrically and aerodynamically and found to produce favorable results. The aerodynamic evaluation proved most encouraging, as an analysis of the USAF/AFAPL HTF compressor predicted an efficiency which was 1 point below experimental results, whereas the original design utilizing the old shock model was about 5 points low in efficiency as compared to the experimental results.

It must be noted that because the model is based on the shock configuration present at the maximum efficiency operating point, the results must be evaluated in that light.

The code has been included in the most recent version of UD0300. The code and user's manual are available upon request from AFWAL/POTX.

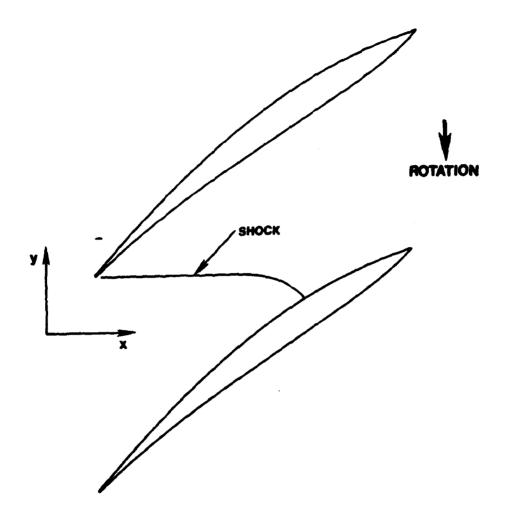


Figure 1. Passage Shock Orientation at Design Operating Point

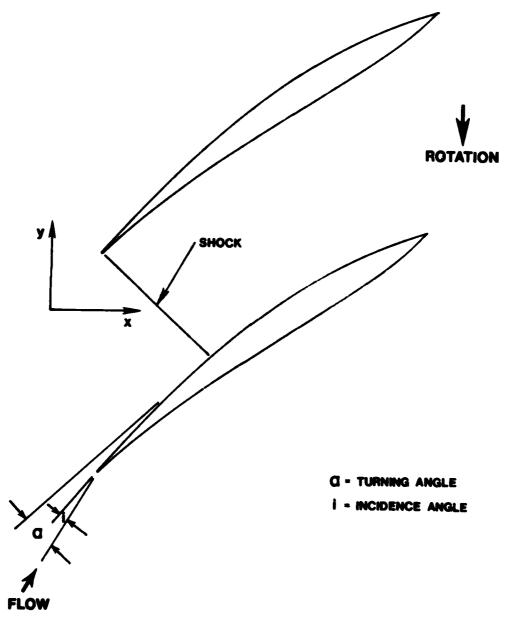


Figure 2. Passage Shock Orientation at the Maximum Efficiency Operating Point

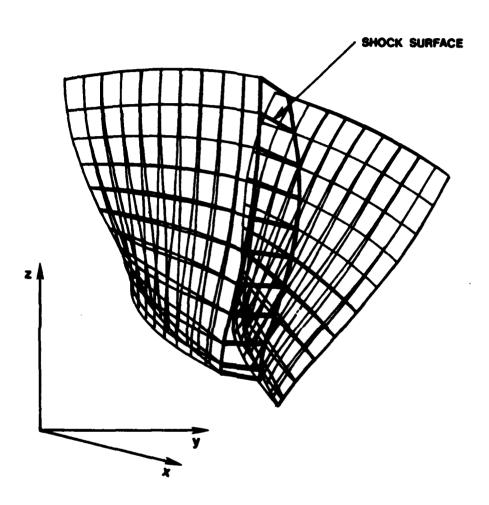
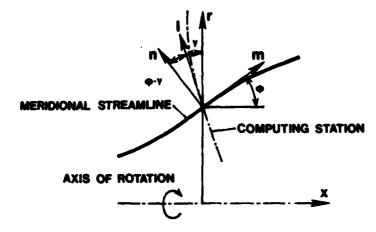
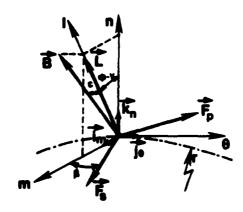


Figure 3. Three-Dimensional View of the Shock Surface



a) m-n-8 system relative to r-x system



b) m-n-0 system

Figure 4. Coordinate Systems

TABLE 1
RESULTS OF TEST CASE I

LEADING EDGE VALUES (DEGREES)

SL	BETA	EPSILON	PHI	GAMMA	NU
1	48.02	0.16	0.00	-0.01	0.12
2	49.67	0.17	0.10	-0.01	0.18
3	51.69	0.18	0.16	-0.01	0.23
4	53.88	0.19	0.19	0.00	0.26
5	56.09	0.19	0.23	-0.01	0.28
6	58.22	0.18	0.25	0.00	0.28
7	60.20	0.18	0.24	0.00	0.27
8	62.02	0.19	0.21	0.00	0.27
9	63.67	0.21	0.16	0.00	0.26
10	65.16	0.27	0.09	0.00	0.28
11	66.50	0.30	0.00	0.00	0.27
	SHOO	K INTERSECTION	N VALUES (D	EGREES)	
SL	ВЕТА	EPSILON	PHI	GAMMA	NU
1	43.80	-10.98	0.00	14.97	3.18
2	45.46	- 9.40	0.14	14.56	3.60
3	47.50	- 6.91	0.24	13.63	4.28
4	49.68	- 4.57	0.33	12.64	4.90
5	51.90	- 2.41	0.38	11.49	5.42
6	54.02	- 0.56	0.41	10.33	5.84
7	56.01	0.99	0.40	9.22	6.17
8	57.84	2.28	0.35	8.20	6.45
9	59.48	3.35	0.27	7.32	6.71
			• • • • • • • • • • • • • • • • • • • •		
10	60.94	4.25	0.15	6.47	6.91

SL = Streamline Number

62.22

11

BETA = Relative Flow Angle EPSILON = Blade Lean Angle

PHI = Stream Surface Pitch Angle

GAMMA = Lean Angle in the Meridional Plane

4.97

NU = Aerodynamic Sweep Angle

0.00

6.04

7.19

TABLE 2
RESULTS OF TEST CASE II

LEADING EDGE VALUES (DEGREES)							
SL	BETA	EPSILON	PHI	GAMMA	NU		
1	48.51	2.82	0.00	30.00	21.26		
2	50.08	2.76	0.27	30.00	20.80		
3	52.00	2.74	0.49	29.97	20.14		
4	54.00	2.64	0.64	30.10	19.36		
5	56.18	2.57	0.70	29.95	18.40		
6	58.19	2.52	0.69	29.91	17.47		
7	60.05	2.47	0.63	30.08	16.67		
8	61.73	2.40	0.51	29.98	15.77		
9	63.21	2.29	0.36	30.01	14.98		
10	64.50	2.15	0.19	30.00	14.22		
11	65.61	2.00	0.00	30.00	13.53		

	SHO	CK INTERSECTION	ON VALUES	(DEGREES)	
SL	BETA	EPSILON	PHI	GAMMA	NU
1	44.96	-13.33	0.00	39.73	18.27
2	46.49	-11.25	0.20	39.60	18.82
3	48.37	- 7.98	0.39	39.21	19.82
4	50.35	- 4.92	0.56	38.82	20.62
5	52.35	- 2.08	0.68	38.12	21.11
6	54.25	0.04	0.69	37.50	21.42
7	56.03	2.34	0.63	37.05	21.60
8	57.66	3.87	0.51	36.42	21.49
9	59.12	5.01	0.36	35.96	21.30
10	60.44	5.80	0.19	35.49	20.96
11	61.63	6.49	0.00	35.25	20.72

TABLE 3
RESULTS OF TEST CASE III

LEADING EDGE VALUES (DEGREES)						
SL	BETA	EPSILON	PHI	GAMMA	NU	
1	48.02	30.53	0.00	-0.01	22.19	
2	50.11	28.71	0.10	-0.01	21.69	
3	52.59	26.90	0.16	-0.01	21.15	
4	55.18	25.13	0.20	-0.00	20.51	
5	57.71	24.45	0.23	-0.01	19.78	
6	60.06	21.91	0.25	0.00	18.98	
7	62.20	20.56	0.24	0.00	18.21	
8	64.11	19.29	0.21	0.00	17.38	
9	65.81	18.14	0.16	0.00	16.57	
10	67.31	17.35	0.09	0.00	16.00	
11	68.64	16.54	0.00	0.00	15.37	

	SHOO	K INTERSECTIO	n values (	D <b>e</b> grees)	
SL	BETA	EPSILON	PHI	GAMMA	NU
1	43.80	18.65	0.00	19.48	25.97
2	45.64	22.80	0.13	18.58	28.13
3	47.78	28.27	0.24	16.61	30.70
4	49.98	33.38	0.32	14.72	33.40
5	52.10	38.05	0.38	12.79	36.05
6	54.09	42.08	0.41	11.03	38.54
7	55.91	45.54	0.40	9.48	40.86
8	57.54	48.40	0.35	8.17	42.90
9	59.01	50.76	0.27	7.10	44.71
10	60.30	52.96	0.15	6.16	46.44
11	61.42	54.69	0.00	5.68	47.97

TABLE 4

RESULTS OF TEST CASE IV

LEADING EDGE VALUES (DEGREES)							
SL	BETA	EPSILON	PHI	GAMMA	NU		
1	48.51	35.71	0.00	30.00	41.67		
2	50.54	33.86	0.27	30.00	40.78		
3	52.93	32.05	0.49	29.97	39.64		
4	55.42	30.25	0.63	30.09	38.28		
5	57.84	28.49	0.70	29.95	36.63		
6	60.07	26.89	0.69	29.91	34.97		
7	62.07	25.52	0.63	30.08	33.46		
8	63.82	24.13	0.51	29.98	31.83		
9	65.34	22.84	0.35	30.01	30.32		
10	66.63	21.84	0.18	30.00	29.05		
11	67.73	20.79	0.00	30.00	27.79		
	SHO	CK INTERSECTION	ON VALUES (	DEGREES)			
SL	ВЕТА	EPSILON	PHI	GAMMA	NU		
1	44.96	22.61	0.00	42.47	40.43		
2	46.68	27.83	0.20	42.05	42.28		
3	48.66	34.38	0.39	41.04	44.70		
4	50.65	40.16	0.56	40.12	47.13		
5	52.56	45.11	0.68	38.98	49.40		
6	54.31	49.18	0.69	37.99	51.53		
7	55.91	52.58	0.63	37.28	53.49		

0.51

0.35

0.19

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36.47

35.88

35.32

35.06

57.34

58.64

59.80

60.84

55.19

57.24

58.98

60.39

8

9

10

11

55.12

56.51

57.75

58.83

### APPENDIX A

## DETAILED DERIVATION OF AERODYNAMIC SWEEP ANGLE

V defined as the angle between  $\vec{F}_S$  and  $\vec{B}$  minus 90°  $\vec{B}$  = cos ε sin (β-γ) $\vec{I}$  - sin ε  $\vec{j}$  + cos ε cos (β-γ) $\vec{k}$   $\vec{F}_S$  = cos β $\vec{I}$  + sin β $\vec{j}$   $\vec{F}_S$  •  $\vec{B}$  =  $|F_S| |B| \cos (90+r)$ But  $|F_S| = |B| = \frac{1}{2}$ ∴  $v = \cos^{-1}(|F_S| |B|) - 90$   $\vec{F}_S$ • $\vec{B}$  = (cos β $\vec{I}$  + sin β $\vec{j}$ )•(cos ε sin(β-γ) $\vec{I}$  - sin ε $\vec{j}$  + cos ε cos (β-γ) $\vec{k}$ )  $|\vec{F}_S$ • $\vec{B}$ |= cos β cos ε sin (β-γ) - sin β sin ε  $v = \cos^{-1}(\cos \beta \cos \epsilon \sin (\beta - \gamma) - \sin \beta \sin \epsilon) - 90$ 

#### REFERENCES

- 1. D. C. Prince, Jr., "Three-Dimensional Shock Structures for Transonic/ Supersonic Compressor Rotors," Journal of Aircraft, Vol. 17, No. 1, January 1980.
- 2. R. M. Hearsey, <u>A Revised Computer Program for Axial Compressor Design</u>, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, ARL-TR-75-0001, Vols. I and II, January 1975.
- 3. A. J. Wennerstrom and W. A. Buzzel, Redesign of a Rotor for a 1500 Ft/Sec, Transonic, High-Through-Flow, Single-Stage, Axial-Flow Compressor With Low Hub/Tip Ratio, Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio, AFAPL-TR-79-2078, September 1979.